

Cosmological Constraints in SUSY with Yukawa Unification

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Abstract. The cosmological relic density of the lightest supersymmetric particle of the minimal supersymmetric standard model is calculated under the assumption of gauge and Yukawa coupling unification. We employ radiative electroweak breaking with universal boundary conditions from gravity-mediated supersymmetry breaking. Further constraints are imposed by the experimental bounds on the b -quark mass and the $BR(b \rightarrow s\gamma)$. We find that coannihilation of the lightest supersymmetric particle, which turns out to be an almost pure bino, with the next-to-lightest supersymmetric particle (the lightest stau) is crucial for reducing its relic density to an acceptable level.

1. Introduction

In recent years the consideration of exotic dark matter has become necessary in order to close the Universe [1]. In the currently favored supersymmetric (SUSY) extensions of the standard model, the most natural WIMP and candidate for CDM is the LSP, i.e. the lightest supersymmetric particle. In the most favored scenarios the LSP is the lightest neutralino, which can be simply described as a Majorana fermion, a linear combination of the neutral components of the gauginos and Higgsinos [1, 10]. Its stability is guaranteed by imposing R-parity conservation, which implies that the LSP's can disappear only by annihilating in pairs.

The simplest and most restrictive version of MSSM with gauge coupling unification is based on the assumption of radiative electroweak symmetry breaking with universal boundary conditions from gravity-mediated soft SUSY breaking. An interesting question is whether this scheme is compatible with exact “asymptotic” unification of the three third family Yukawa couplings. A positive answer to this question would be very desirable since it would lead to a simple and highly predictive theory.

In this presentation we summarize the results of Refs. [3, 4], aimed to answer the question of whether the simple version of the MSSM with Yukawa unification at the GUT scale, can be compatible with the most restrictive phenomenological constraints, which are the correct predictions for b -quark mass and $BR(b \rightarrow s\gamma)$, and satisfy the requirement that the relic abundance $\Omega_{LSP} h^2$ of the lightest

supersymmetric particle (LSP) in the universe does not exceed the upper limit on the cold dark matter (CDM) abundance implied by cosmological considerations.

2. Input parameters on the MSSM with Yukawa Unification

We consider the MSSM embedded in some general supersymmetric GUT based on a gauge group such as $SO(10)$ or E_6 (where all the particles of one family belong to a single representation) with the additional requirement that the top, bottom and tau Yukawa couplings unify [2] at the GUT scale M_{GUT} . Ignoring the Yukawa couplings of the first and second generation, the effective superpotential below M_{GUT} is

$$W = \epsilon_{ij}(-h_t H_2^i Q_3^j t^c + h_b H_1^i Q_3^j b^c + h_\tau H_1^i L_3^j \tau^c + \mu H_1^i H_2^j) , \quad (1)$$

where $Q_3 = (t, b)$ and $L_3 = (\nu_\tau, \tau)$ are the quark and lepton $SU(2)_L$ doublet left handed superfields of the third generation and t^c , b^c and τ^c the corresponding $SU(2)_L$ singlets. Also, H_1 , H_2 are the electroweak higgs superfields and $\epsilon_{12} = +1$. The gravity-mediated soft supersymmetry breaking terms in the scalar potential are given by

$$V_{soft} = \sum_{a,b} m_{ab}^2 \phi_a^* \phi_b + \left(\epsilon_{ij}(-A_t h_t H_2^i \tilde{Q}_3^j \tilde{t}^c + A_b h_b H_1^i \tilde{Q}_3^j \tilde{b}^c + A_\tau h_\tau H_1^i \tilde{L}_3^j \tilde{\tau}^c + B \mu H_1^i H_2^j) + h.c. \right) , \quad (2)$$

where the ϕ_a 's are the (complex) scalar fields and tildes denote superpartners. The gaugino mass terms in the Lagrangian are

$$\frac{1}{2}(M_1 \tilde{B} \tilde{B} + M_2 \sum_{r=1}^3 \tilde{W}_r \tilde{W}_r + M_3 \sum_{a=1}^8 \tilde{g}_a \tilde{g}_a + h.c.) , \quad (3)$$

where \tilde{B} , \tilde{W}_r and \tilde{g}_a are the bino, winos and gluinos respectively. ‘Asymptotic’ Yukawa coupling unification implies

$$h_t(M_{GUT}) = h_b(M_{GUT}) = h_\tau(M_{GUT}) \equiv h_0 . \quad (4)$$

Based on $N = 1$ supergravity, we take universal soft supersymmetry breaking terms at M_{GUT} , i.e., a common mass for the scalar fields m_0 , a common trilinear scalar coupling A_0 and $B_0 = A_0 - m_0$. Also, a common gaugino mass $M_{1/2}$ is assumed at M_{GUT} .

Our effective theory below M_{GUT} depends on the parameters ($\mu_0 = \mu(M_{GUT})$)

$$m_0, M_{1/2}, A_0, \mu_0, \alpha_G, M_{GUT}, h_0, \tan \beta .$$

α_G and M_{GUT} are evaluated consistently with the experimental values of α_{em} , α_s and $\sin^2 \theta_W$ at m_Z . We integrate numerically the renormalization group equations (RGEs) for the MSSM at two loops in the gauge and Yukawa couplings from M_{GUT} down to a common supersymmetry threshold $M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$. From this energy to m_Z , the RGEs of the nonsupersymmetric standard model are used.

$\tan\beta$ is estimated at the scale M_S using the experimental input $m_\tau(m_\tau) = 1.777$ GeV. We incorporate the SUSY threshold correction to $m_\tau(M_S)$ from the approximate formula of Ref.[14]. It is about 8%, for $\mu > 0$, leading to a value of $\tan\beta = 55.4 - 54.5$ for $m_A = 100 - 700$ GeV, while, for $\mu < 0$, we find a correction of about -7% and $\tan\beta = 47.8 - 46.9$ in the same range of m_A .

h_0 is found by fixing the top quark mass at the center of its experimental range, $m_t(m_t) = 166$ GeV. The value obtained for $m_b(m_Z)$ after including supersymmetric corrections is somewhat higher than the experimental limit.

A_0 , for simplicity we take $A_0 = 0$. Our results move very little for negative values of A_0 bigger than about $-5M_{1/2}$, however lower negative values and positive values of this parameter tend to increase the SUSY spectrum increasing $\Omega_{LSP} h^2$. Therefore the limits on $\Omega_{LSP} h^2$ imposes lower and upper bounds on A_0 .

$m_0, M_{1/2}, \mu_0$, As we will discuss later the electroweak symmetry breaking, when Yukawa unification is assumed, imposes a relation on the values of m_0 and $M_{1/2}$. On the other hand the role of coannihilation $\tilde{\chi}-\tilde{\tau}$ make convenient to express our results in terms of relative mass splitting $\Delta_{\tilde{\tau}_2} = (m_{\tilde{\tau}_2} - m_{\tilde{\chi}})/m_{\tilde{\chi}}$ between the NLSP and LSP. Therefore we trade the GUT values of $m_0, M_{1/2}$ and μ_0 by the pseudoscalar Higgs mass m_A and $\Delta_{\tilde{\tau}_2}$.

Let us describe with more detail the last item above. Assuming radiative electroweak symmetry breaking, we can express the values of the parameters μ (up to its sign) and B at M_S in terms of the other input parameters by means of the appropriate conditions

$$\mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2}m_Z^2, \quad \sin 2\beta = -\frac{2B\mu}{m_{H_1}^2 + m_{H_2}^2 + 2\mu^2}, \quad (5)$$

where m_{H_1}, m_{H_2} are the soft supersymmetry breaking scalar higgs masses. When unified Yukawa couplings and a common value for m_{H_2} and m_{H_1} are assumed GUT, we find both $m_{H_2}^2$ and $m_{H_1}^2$ negative at M_S . However for certain values of m_0 and $M_{1/2}$ is possible to find values the pseudoscalar Higgs,

$$m_A = m_{H_1}^2 + m_{H_2}^2 + 2\mu^2$$

beyond a lower bound, considered to be m_Z in the present work. Furthermore, the authors of Ref. [6] found that, for every value of m_A and a fixed value of $m_t(m_t)$, there is a pair of minimal values of m_0 and $M_{1/2}$ where the masses of the LSP and $\tilde{\tau}_2$ are equal. This is understood from the dependence of m_A on m_0 and $M_{1/2}$ given in Ref. [5]:

$$m_A^2 = \alpha M_{1/2}^2 - \beta m_0^2 - \text{const.}, \quad (6)$$

where all the coefficients are positive and α and β , which depend only on $m_t(m_t)$, are ~ 0.1 (the constant turns out to be numerically close to m_Z^2). Equating the masses of the LSP and $\tilde{\tau}_2$ is equivalent to relating m_0 and $M_{1/2}$. Then, for every

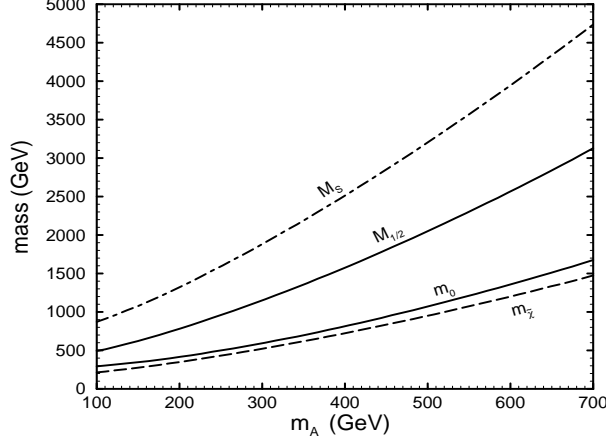


Figure 1. The values of $m_{\tilde{\chi}}$, m_0 , $M_{1/2}$ and M_S as functions of m_A for $\mu > 0$, $A_0 = 0$ and $m_{\tilde{\tau}_2} = m_{\tilde{\chi}}$. These values are affected very little by changing the sign of μ .

m_A , a pair of values of m_0 and $M_{1/2}$ is determined. We had included the full one-loop radiative corrections to the effective potential as given in the appendix E of Ref. [14].

The values of the LSP, M_S and the corresponding values of m_0 and $M_{1/2}$ are given in Fig.1.

3. Phenomenological constraints from m_b and $b \rightarrow s\gamma$

A significant problem, which may be faced in trying to reconcile Yukawa unification and universal boundary conditions, is due to the generation of sizeable SUSY corrections to the b -quark mass [5, 7]. The sign of these corrections is opposite to the sign of the MSSM parameter μ (with the conventions of Ref.[3]). As a consequence, for $\mu < 0$, the tree-level value of m_b , which is predicted from Yukawa unification already near its experimental upper bound, receives large positive corrections which drive it well outside the allowed range. However, it should be noted that this problem arises in the simplest realization of this scheme. In complete models correctly incorporating fermion masses and mixing, m_b can receive extra corrections which may make it compatible with experiment. So, we do not consider this b -quark mass problem absolutely fatal for the $\mu < 0$ case. However in the alternative scenario, with $\mu > 0$, the b -quark mass receives negative SUSY corrections and can easily be compatible with data in this case. An example of the typical values we find is given in table I.

This scheme with $\mu > 0$, is severely restricted by the recent experimental results [8] on the inclusive decay $b \rightarrow s\gamma$ [9]. It is well-known that the SUSY corrections to the inclusive branching ratio $\text{BR}(b \rightarrow s\gamma)$, in the case of the MSSM with universal boundary conditions, arise mainly from chargino loops and have the same sign with the parameter μ . Consequently, these corrections interfere constructively with the contribution from the standard model (SM) including

-----	4.5 GeV Prediction $\mu < 0$
-----	3.41 GeV tree level value $\mu < 0$
=====	3.17 GeV exp. bound
=====	3.13 tree level value $\mu > 0$
=====	2.3 GeV Prediction $\mu > 0$
=====	Exp. bound 2.17 GeV

Table I. Values for the quark bottom mass.

an extra electroweak Higgs doublet. However, this contribution is already bigger than the experimental upper bound on $\text{BR}(b \rightarrow s\gamma)$ for not too large values of the CP-odd Higgs boson mass m_A . As a result, in the present context with Yukawa unification and hence large $\tan\beta$, a lower bound on m_A is obtained for $\mu > 0$. On the contrary, for $\mu < 0$, the SUSY corrections to $\text{BR}(b \rightarrow s\gamma)$ interfere destructively with the SM plus extra Higgs doublet contribution yielding, in most cases, no restrictions on the parameters. The results corresponding to the parameters given in fig.1 are shown in fig.2. In the case of $\mu > 0$, $\text{BR}(b \rightarrow s\gamma)$ decreases as the splitting between $m_{\tilde{\tau}_2}$ and $m_{\tilde{\chi}}$ increases.

4. LSP relic abundance and bino–stau coannihilations

The cosmological constraint on the parameter space results from the requirement that the relic abundance $\Omega_{LSP} h^2$ of the lightest supersymmetric particle (LSP) in the universe does not exceed the upper limit on the cold dark matter (CDM) abundance implied by cosmological considerations (Ω_{LSP} is the present energy density of the LSPs over the critical energy density of the universe and h is the present value of the Hubble constant in units of $100 \text{ km sec}^{-1} \text{ Mpc}^{-1}$). Taking both the currently available cosmological models with zero/nonzero cosmological constant, which provide the best fits to all the data, as equally plausible alternatives for the composition of the energy density of the universe and accounting for the observational uncertainties, we obtain the restriction $\Omega_{LSP} h^2 \lesssim 0.22$ (see Refs.[3]). Assuming that all the CDM in the universe is composed of LSPs, we further get $\Omega_{LSP} h^2 \gtrsim 0.09$.

The cosmological relic density of the lightest neutralino $\tilde{\chi}$ (almost pure \tilde{B}) in MSSM with Yukawa unification increases to unacceptably high values as $m_{\tilde{\chi}}$ becomes larger. Low values of $m_{\tilde{\chi}}$ are obtained when the NLSP ($\tilde{\tau}_2$) is almost degenerate with $\tilde{\chi}$. Under these circumstances, coannihilation of $\tilde{\chi}$ with $\tilde{\tau}_2$ and $\tilde{\tau}_2^*$ is of crucial importance reducing further the $\tilde{\chi}$ relic density by a significant amount. The important role of coannihilation of the LSP with sparticles carrying masses close to its mass in the calculation of the LSP relic density has been pointed out by many authors (see e.g., Refs.[12, 13, 11]). Here, we will use the method described by Griest and Seckel [11]. Note that our analysis can be

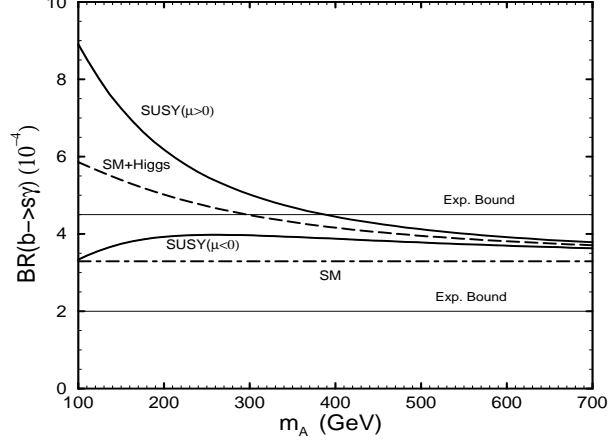


Figure 2. The central value of the SUSY inclusive $\text{BR}(b \rightarrow s\gamma)$ as function of m_A for both signs of μ , $A_0 = 0$ and $m_{\tilde{\tau}_2} = m_{\tilde{\chi}}$. The contributions from the SM and the SM plus charged Higgs boson (SM+Higgs) as well as the experimental bounds on $\text{BR}(b \rightarrow s\gamma)$, 2×10^{-4} and 4.5×10^{-4} , are also indicated.

readily applied to any MSSM scheme where the LSP and NLSP are the bino and stau respectively.

The relic abundance of the LSP at the present cosmic time can be calculated from the equation:

$$\Omega_{\tilde{\chi}} h^2 \approx \frac{1.07 \times 10^9 \text{ GeV}^{-1}}{g_*^{1/2} M_P x_F^{-1} \hat{\sigma}_{eff}} \quad (7)$$

with

$$\hat{\sigma}_{eff} \equiv x_F \int_{x_F}^{\infty} \langle \sigma_{eff} v \rangle x^{-2} dx . \quad (8)$$

Here $M_P = 1.22 \times 10^{19} \text{ GeV}$ is the Planck scale, $g_* \approx 81$ is the effective number of massless degrees of freedom at freeze-out and $x_F = m_{\tilde{\chi}}/T_F$, with T_F being the freeze-out photon temperature.

In our case, σ_{eff} takes the form

$$\sigma_{eff} = \sigma_{\tilde{\chi}\tilde{\chi}^*} r_{\tilde{\chi}} r_{\tilde{\chi}} + 4\sigma_{\tilde{\chi}\tilde{\tau}_2} r_{\tilde{\chi}} r_{\tilde{\tau}_2} + 2(\sigma_{\tilde{\tau}_2\tilde{\tau}_2} + \sigma_{\tilde{\tau}_2\tilde{\tau}_2^*}) r_{\tilde{\tau}_2} r_{\tilde{\tau}_2} . \quad (9)$$

For r_i , we use the nonrelativistic approximation

$$r_i(x) = \frac{g_i(1 + \Delta_i)^{3/2} e^{-\Delta_i x}}{g_{eff}} , \quad (10)$$

$$g_{eff}(x) = \sum_i g_i(1 + \Delta_i)^{3/2} e^{-\Delta_i x}, \quad \Delta_i = (m_i - m_{\tilde{\chi}})/m_{\tilde{\chi}} . \quad (11)$$

Here $g_i = 2, 1, 1$ ($i = \tilde{\chi}, \tilde{\tau}_2, \tilde{\tau}_2^*$) is the number of degrees of freedom of the particle species i with mass m_i and $x = m_{\tilde{\chi}}/T$ with T being the photon temperature.

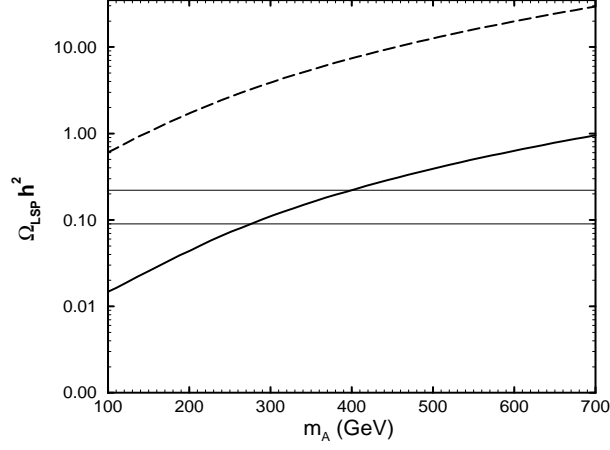


Figure 3. The LSP relic abundance $\Omega_{LSP} h^2$ as function of m_A in the limiting case $m_{\tilde{\tau}_2} = m_{\tilde{\chi}}$ and for $\mu > 0$, $A_0 = 0$. The solid line includes coannihilation of $\tilde{\tau}_2$ and $\tilde{\chi}$, while the dashed line is obtained by only considering the LSP annihilation processes. These results are affected very little by changing the sign of μ . The limiting lines at $\Omega_{LSP} h^2 = 0.09$ and 0.22 are also included.

The freeze-out temperatures which we obtain here are of the order of $m_{\tilde{\chi}}/25$ and, thus, our nonrelativistic approximation (see Eq.(10)) is justified. Under these circumstances, the quantities $\sigma_{ij}v$ are well approximated by their Taylor expansion up to second order in the ‘relative velocity’,

$$\sigma_{ij}v = a_{ij} + b_{ij}v^2. \quad (12)$$

The thermally averaged cross sections are then easily calculated

$$\langle \sigma_{ij}v \rangle(x) = \frac{x^{3/2}}{2\sqrt{\pi}} \int_0^\infty dv v^2 (\sigma_{ij}v) e^{-xv^2/4} = a_{ij} + 6b_{ij}/x. \quad (13)$$

The contribution of the a_{ij} ’s to the cross section is more important than the b_{ij} ’s since its contribution is suppressed by a factor $6/x_F \approx .2$. The annihilation cross section is suppressed respect the coannihilation channels due to the fact that $a_{\tilde{\chi}\tilde{\chi}}$ is suppressed due to Fermi statistics [10]. This suppression is not present in the coannihilation channels, however its contribution to σ_{eff} is attenuated by the exponential in $r_{\tilde{\tau}_2}$ in eq. (10). Therefore coannihilations have an important effect in decreasing the Ω_{LSP} when the LSP and the NLSP are very close in mass. In our case we find this effect negligible when the mass splitting $\tilde{\chi}-\tilde{\tau}$ is greater than approximately 20%. The complete list of Feynman diagrams and the expressions for the a_{ij} ’s appropriate for large $\tan\beta$ are given in Ref. [3].

The result of including coannihilation of $\tilde{\tau}_2$ and $\tilde{\chi}$ in the computation of the LSP relic abundance is clearly shown in fig.3. The effect of increase the mass splitting between $\tilde{\tau}_2$ and $\tilde{\chi}$ will result in larger values for Ω_{LSP} .

5. Conclusions

We have shown that the condition of “asymptotic” Yukawa unification on the MSSM results in a significant constraint on the free parameter space of the model.

Constraints from m_b and $b \rightarrow s\gamma$ can be simultaneously satisfied for $\mu > 0$ and relatively large values of the SUSY parameters $m_A \sim 385\text{GeV}$, $m_{\tilde{\chi}} \sim 695\text{GeV}$, $m_0 \sim 780\text{GeV}$ and $M_{1/2} \sim 1.5\text{TeV}$. The constraint derived from cosmological limits on LSP relic abundance, is satisfied only when bino–stau coannihilations are relevant. If we superpose figs. 1, 2, 3 we can observe that the previous conditions are satisfied in a narrow band of parameter space.

The high values of $m_{\tilde{\chi}}$ in the parameter space allowed by the constraints described above will make difficult its direct detection. However the large $\tan\beta$ scheme can provide interesting predictions [15] if one relaxes the strict unification condition imposed in the present study.

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